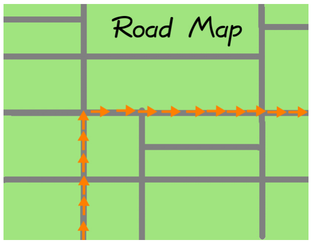
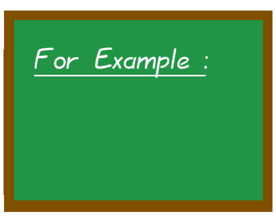
Module 1.4: Advanced Venn Diagram Problems



In this module, we'll consider some harder Venn Diagram problems. First, we're going to learn about an alternative format for displaying this kind of information: a table rather than a Venn Diagram. These are impractical for 3-variable problems, but for 2-variable problems, they do give you some good information. Second, we will look at 3-variable Venn Diagram problems that require a variable. Third, we will explore "the inclusion-exclusion formula," which will have much value in our work later in this chapter. Fourth, we will learn a great deal about blood types—information that hopefully you won't have to act on in an emergency, but certainly one of the most important applications of set theory.



1-4-1

Suppose that a car manufacturer is trying to predict demand for the next sales cycle, based on the previous year's data. The company database is searched, and it turns out that the luxury sound system was ordered in 2541 models, and the normal sound system was ordered in 7983. The sunroof was ordered in 3048 models, and of those, 2115 ordered the luxury sound system also. Find out how many orders there were for all possible combinations of luxury/normal stereo and with/without sunroof.

We start by filling in the given data

	Sunroof		
Sound System	With	Without	Total
Luxury Sound	2115	??	2541
Normal Sound	??	??	7983
Total	3048	??	??

and then we can see the following:

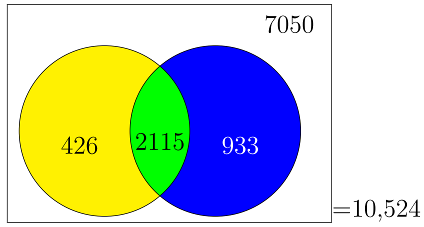
1. The total cars ordered is $2541 + 7983 = 10,524$.
2. Those with the luxury sound system but no sunroof number $2541 - 2115 = 426$.
3. Those with normal sound and a sunroof come to $3048 - 2115 = 933$.
4. Those without a sunroof and with a normal sound system come to $7983 - 933 = 7050$.
5. In general, those without a sunroof are $10,524 - 3048 = 7476$.

The final table is given in the next box.

Here is the final table for the previous example.

	Sunroof		
Sound System	With	Without	Total
Luxury Sound	2115	426	2541
Normal Sound	933	7050	7983
Total	3048	7476	10,524

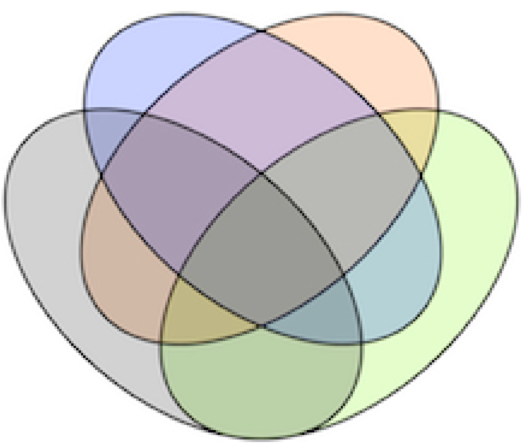
Take a moment to convince yourself that the previous example would have the following Venn Diagram. The left circle represents cars with luxury sound systems, and the right circle represents cars with sunroofs.



We've been having fun with Venn Diagrams up to now, and so you might wonder why one would want to use tables for this sort of problem. It turns out that if it is possible to do a Venn Diagram problem, then it is possible to do the equivalent problem with a table, and vice versa.

For two-variable problems, it is simply a matter of taste. Some students have a strong preference for the Venn Diagram, some students have a strong preference for a table, and most students do not have a strong preference. However, with a three-variable problem, the table becomes somewhat large and difficult to work with.

Last but not least, it is only fair to mention that the tables convey more information at a glance, as you will discover shortly. Therefore, it is good to explore this other approach, at least briefly.



This is what a four-variable Venn Diagram looks like. As you can see, it is made of ellipses (ovals) instead of circles. Each of the 16 regions corresponds to either "in" or "out" for four inter-related sets.

This can be rather dizzying to work with. Luckily, we will not be exploring 4-variable problems in this textbook.

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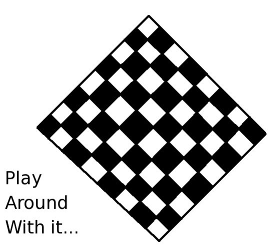


\# 1-4-2

We are going to convert some old Venn Diagrams into a table now.

- Convert the internship data from the problem on Page 106 into a table.
- Convert the international commerce MBA admissions data from the problem on Page 60 into a table.

[Answers: at the end of the module, on Page 144.]

For Example :

1-4-3

Suppose for another car, perhaps in Europe where automatic transmissions are a luxury item (people there prefer to "drive stick," i.e. use a manual transmission), another survey is being done. The engine of this model comes in 4-cylinders and 6-cylinders, and the transmission is either manual or automatic. The basic model, which sold 45,681 times has 4-cylinders and a manual transmission. The four-cylinder models of all kinds sold 58,291 models, and the manual transmission was found in 71,488 cars. The car sold 89,563 models for the year among all options.

Let's put the given data in a table.

	Engine		
Transmission	4-Cylinder	6-Cylinder	Total
Automatic	??	??	??
Manual	45,681	??	71,488
Total	58,291	??	89,563

At first, it might not look like we have enough data to find everything, but let's try and see what happens. We will continue in the next box.

Continuing with the previous box,

1. The number of 6-cylinder sales in total would be $89,563 - 58,291 = 31,272$.
2. The number of 6-cylinder manuals would be $71,488 - 45,681 = 25,807$.
3. The number of automatic 4-cylinders is therefore $58,291 - 45,681 = 12,610$.
4. The number of automatic 6-cylinders is therefore $31,272 - 25,807 = 5465$.
5. Finally, the total number of automatics is $12,610 + 5465 = 18,075$.
6. The final table is:

	Engine		
Transmission	4-Cylinder	6-Cylinder	Total
Automatic	12,610	5465	18,075
Manual	45,681	25,807	71,488
Total	58,291	31,272	89,563

We can also make some quick checks of our work now, such as verifying the grand total:

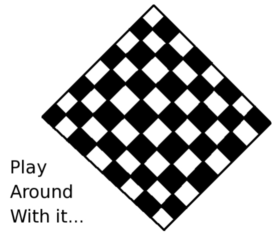
$$18,075 + 71,488 = 89,563$$

Play Around With it...

1-4-4

Suppose that you're commissioned to do research for an ice-cream company. There is a survey of 100 customers. Of these, 50 like vanilla, 60 like chocolate, and 40 like both. Make a table as was done in the previous box.

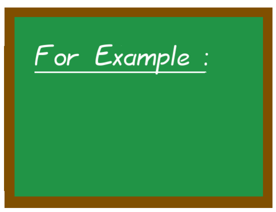
The solution is given at the end of the module on Page 145.



1-4-5

Continuing with the previous box, it seems there was an error in the collected data. Actually, only 10 people like both flavors of ice-cream. Make another table.

The solution is given at the end of the module on Page 145.



1-4-6

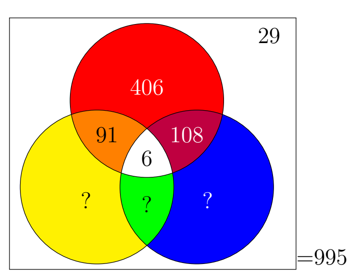
Suppose we are classifying mushrooms from *Nine-hundred and Ninety Five Popular Mycophile's Mushrooms* while entering them into a database. We will use a three-variable Venn Diagram. Let the top circle indicate those mushrooms that are lethal. Let the left circle indicate those mushrooms that are tasty. Let the right circle indicate those mushrooms that are spotted. There are six species in the book which are spotted, lethal, and tasty. The total number of tasty mushrooms is 345. The total number of spotted mushrooms is 263, and the total number of lethal mushrooms is 611. There are 29 mushrooms which are neither spotted, nor lethal, nor tasty. If 114 are spotted and lethal, and 91 are lethal and tasty but not spotted, how many are tasty and spotted but not lethal?

This one is tricky, so we'll go with a step-by-step analysis, given in the next box.

These are the steps for solving the problem in the previous box.

1. We know 6 mushrooms belong in the triangular bit in the center, so we write them in immediately.
2. Likewise, the 29 that are in the "no category" go into the background.
3. Then also the 91 that are lethal and tasty but not spotted should go in the upper-left-hand mitre-shaped bit.
4. There are 114 that are both spotted and lethal. Since 6 are in all three categories, that means that $114 - 6 = 108$ are spotted and lethal but not tasty. Of course, they belong in the upper-right-hand mitre-shaped bit.
5. Now the total number of lethal mushrooms is given as 611. If we subtract the three filled-in parts of the top circle, we learn that
$$611 - 108 - 6 - 91 = 406$$
are lethal but neither tasty nor spotted.
6. Of course, the universal set is the set of 995 mushrooms; we know that from the title of the book.
7. Now, we appear to be stuck. Take a moment to verify that we cannot go further.

Here is the Venn Diagram, as it stands after completing the steps of the previous box, without continuing on to the steps of the next box.



Continuing with the problem of the previous three boxes…

8. Let's put an x on the set we need to tabulate, which is the tasty and spotted but not lethal part, or the mitre-shaped bit between the left circle and the right circle.

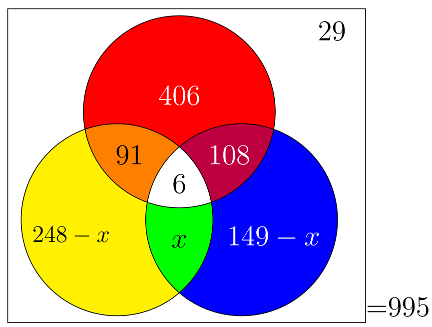
Note: By the way, I have chosen this specific spot for the x, not only because that spot is not yet known, but it is as close to the center as possible.

9. The left circle, in total, is to cover 345 tasty mushrooms. We've accounted for 91, 6, and x in the small parts, so the large part of the circle should have $345 - 91 - 6 - x$, which can be written as $248 - x$.

10. Likewise, the right circle, in total, is to cover 263 spotted mushrooms. We've accounted for 108, 6, and x in the small parts, so the large part of the circle should have $263 - 108 - 6 - x$, which can be written $149 - x$.

Note: Another way to look at why we chose to place the x in the place where we did is to look at it this way: if we mark that middle region as x, then we do not need any y or any second variable, because the other two empty regions can be written as $248 - x$ and as $149 - x$.

At this point, the Venn Diagram should look like this:



Continuing with the problem of the previous five boxes…

11. Now, let's add everything up, and set it equal to 995.

$$\begin{aligned}(248 - x) + x + 6 + 91 + 108 + (149 - x) + 406 + 29 &= 995\\ (248 + 6 + 91 + 108 + 149 + 406 + 29) - x - x + x &= 995\\ 1037 - x &= 995\\ x &= 42\end{aligned}$$

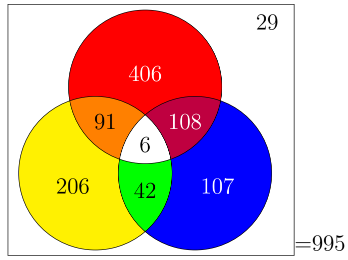
12. Therefore, the answer is 42. There are 42 mushrooms that are spotted and tasty, but not lethal.

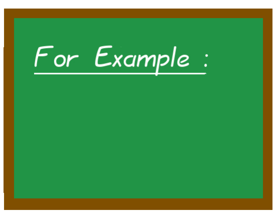
13. Since we've gotten this far, we should complete the diagram. The $149 - x$ becomes 107 because $149 - 42 = 107$.

14. Likewise the $248 - x$ becomes 206 because $248 - 42 = 206$.

The final Venn Diagram is given in the next box.

This is the final Venn Diagram for the mushroom problem, of the last six boxes.





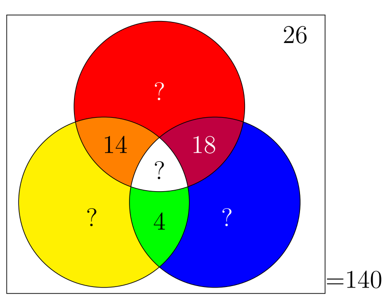
1-4-7

I think you'll enjoy this problem. It is a great puzzle and involves some healthy mental calisthenics. Suppose that a fairly large company on Oahu is having an all-day conference on some new video-conferencing tool. The Chief Learning Officer (CLO) of the company decides to get pizza for all those attending the 140-person conference. He sends an intern to ask about toppings during the 10 AM coffee break. The intern finds out that 70 attendees like pepperoni, 46 like pineapple, 52 like onions, and 26 like a plain pizza (one with no toppings). He adds that "lots of people would like more than one topping." Sadly, the intern didn't record how many people would like more than one topping.

At the 11 AM coffee break, the intern is sent back into the conference hall. He records that 14 attendees like pineapple and pepperoni but not onions, 18 attendees like pepperoni and onions but not pineapple, but only 4 like pineapple and onions but not pepperoni. He finishes with "and a significant number liked all three." The CLO asks what he means by "a significant number," and the intern shrugs, replying "a bunch." It is now 11:15 AM and the pizzas have to be ordered by 11:30 AM, otherwise they won't arrive on time for lunch at noon. There is no coffee break between now and 11:30, so there can be no more questions. Our challenge is to use algebra and Venn Diagrams to summarize the situation, finding out the number who want every possible subset of $\{\text{pepperoni}, \text{onions}, \text{pineapple}\}$. For simplicity, let us agree that pepperonis should be on top, pineapples should be on the left, and onions should be on the right.

We will solve the problem, starting in the next box.

We cannot use the 70, the 46, and the 52, because those represent full circles and four regions. However, the 26 who like plain pizza belong in the background. Then, we can definitely use the three numbers (14, 18, and 4) found out during the second coffee break. Furthermore, looking at the first paragraph, we learn that the conference has 140 attendees. That fills in only five spots out of nine, leaving four blank. We are stuck now. The following Venn Diagram is the summary of where we are at this stage.



We will continue in the next box.

Because we are stuck, let's put an x as close to the center as we can. In this case, the x would go into the very center. Then, each of the complete circles has three-out-of-four areas with an entry, and only one blank. We can now fill in those blank entries. For the pepperoni-only spot, we should insert $38 - x$ so that the complete pepperoni circle comes out to

$$(38 - x) + 14 + x + 18 = 38 + 14 + 18 = 70 \qquad \checkmark$$

Similarly, for the pineapple-only spot, we should insert $28 - x$ so that the complete pineapple circle comes out to

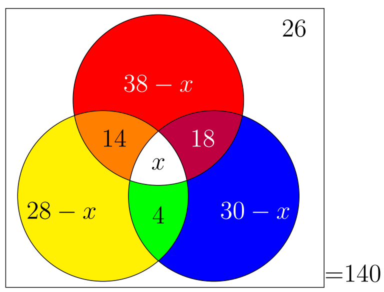
$$(28 - x) + 14 + x + 4 = 28 + 14 + 4 = 46 \qquad \checkmark$$

and likewise for the onions-only spot, we should insert $30 - x$ so that the complete onion circle comes out to

$$(30 - x) + 18 + x + 4 = 30 + 18 + 4 = 52 \qquad \checkmark$$

We can summarize the situation in the next box.

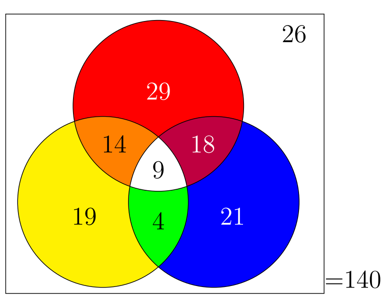
At this stage, we have the following Venn Diagram:



As you can see, all nine spots are filled in, so we can write the equation:

$$\begin{aligned}
26 + (38 - x) + 14 + 18 + x + (28 - x) + 4 + (30 - x) &= 140 \\
(26 + 38 + 14 + 18 + 28 + 4 + 30) + (-3x + x) &= 140 \\
158 - 2x &= 140 \\
18 &= 2x \\
9 &= x
\end{aligned}$$

Now that we know that the center entry of the Venn Diagram is $x = 9$, we can go back and fill in all the entries with numbers. We get the following Venn Diagram



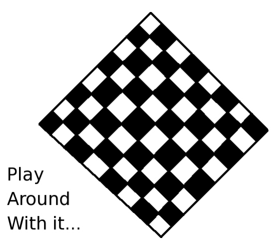
which we will check in the next box.

Let's check our work from the previous few boxes.

- The diagram shows that 14 attendees like pineapple and pepperoni, but not onions. ✓
- The diagram shows that 18 attendees like pepperoni and onions, but not pineapple. ✓
- The diagram shows that 4 attendees like onions and pineapple, but not pepperoni. ✓
- The diagram shows that 26 attendees prefer plain pizza. ✓
- The number of attendees who like pepperoni is $29 + 14 + 9 + 18 = 70$. ✓
- The number of attendees who like pineapple is $19 + 4 + 9 + 14 = 46$. ✓
- The number of attendees who like onions is $18 + 9 + 4 + 21 = 52$. ✓
- Finally, the total number of attendees comes to

$$26 + 29 + 14 + 18 + 9 + 19 + 4 + 21 = 140 \checkmark$$

We appear to be correct! On exams, always remember... while checking your work does take a lot of time, it takes a lot less time than repeating the course!



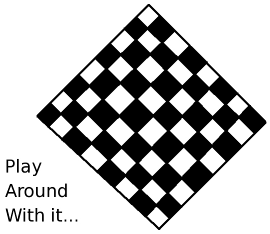
1-4-8

I hope you enjoyed the problem of the last five boxes. As it turns out, that intern has a twin brother, in the CS-recruiting business. Very early in the morning, a bunch of candidates will arrive for a series of interviews throughout the day. The relevant skills are knowledge of SQL, Python, and Java. The intern tells his coworker the following facts:

- 70 candidates know Python.
- 75 candidates know Java.
- 55 candidates know SQL.
- 5 candidates know none of these three languages. What place is there in the company for people who cannot program?!
- 26 candidates know Java and Python but not SQL.
- 15 candidates know Python and SQL but not Java.
- 18 candidates know SQL and Java but not Python.
- There are 108 candidates total.

The coworker just wants to know how many people program in only one language, as well as how that breaks down into SQL-only, Java-only, and Python-only. The candidates will arrive soon, so please figure this out right away.

If you want to compare your Venn Diagram with mine, then you will find my diagram on Page 145. For simplicity of comparison, let's put Python on top, Java on the left, and SQL on the right.



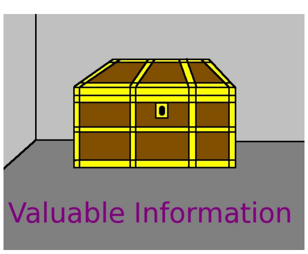
1-4-9

Suppose an NYPD (New York Police Department) precinct is surveying its 147 police officers about what foreign languages they can speak, in order to better figure out who should be assigned to touristy areas. Here is some data.

- 50 officers speak French.
- 38 officers speak Mandarin.
- 68 officers speak Spanish.
- 8 officers speak both French and Mandarin.
- 14 officers speak French and Spanish but not Mandarin.
- 1 officer speaks all three of these languages: French, Spanish, Mandarin.
- 25 officers are unable to speak any of these three languages: French, Spanish, Mandarin.

Summarize the data, and then also tell me how many officers can speak exactly 0, 1, 2, and 3 of these languages. The answers will be given on Page 146.

Because we'll look at the previous problem three times, let's call this "Version 1." One version will actually be impossible, in a strict sense.

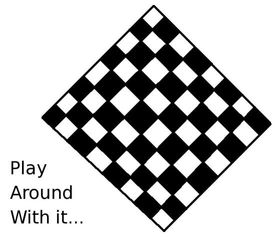


We are now ready to deal with greater precision on the question of impossible Venn Diagrams, which we touched on, back on Page 74. There are three ways an impossible Venn Diagram can occur:

- After solving the problem, and checking your work, you find a negative number in one or more spots. The size of a set is never negative. (We'll see an example of this type in the next box.)
- Not enough facts are given. The true proof of such a situation is that you make two Venn Diagrams that are different from each other, but that both satisfy ALL the given facts. When this occurs, you know that not enough facts were given for you to be able to solve the situation. (We saw one of this type on Page 74.)

Note: The above two are fairly common. The last one is somewhat rare.

- You solve the problem completely without using one of the given facts. You check your solution against the facts that you actually used, and everything matches. However, your final Venn Diagram contradicts this one fact that you did not use. This contradiction means that your solution does not satisfy all the given facts, and furthermore, that no Venn Diagram could do so.

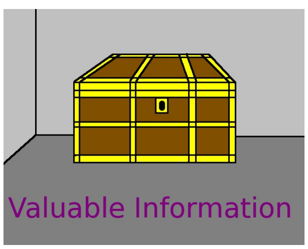


1-4-10

Now I'd like to show you an impossible Venn Diagram problem. Here is Version 2 of the NYPD problem. Suppose an NYPD police precinct is surveying its 163 police officers about what foreign languages they can speak, in order to better figure out who should be assigned to touristy areas.

- 60 officers speak French.
- 38 officers speak Mandarin.
- 68 officers speak Spanish.
- 8 officers speak both French and Mandarin.
- 14 officers speak French and Spanish but not Mandarin.
- 7 officers speak all three of these languages: French, Spanish, Mandarin.
- 25 officers are unable to speak any of these three languages: French, Spanish, Mandarin.
- Warning! This problem will have a solution that is impossible, because of a negative number in one of the spots.

For this problem, construct the Venn Diagram, and then stop. My diagram will be given on Page 146.



The take-away message from the above problem is that, if you see a negative number in a part of a Venn Diagram, then because the size of a set can never be negative, you know that one of the following three things has occurred.

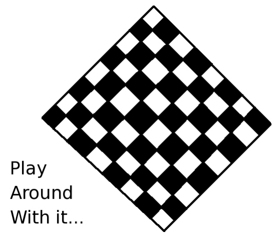
- The data is incorrect—there was a transcription error, a typo, or something like that.
- The situation is actually impossible.
- You have made an arithmetic error.

No problems in this textbook are impossible, unless they are especially marked as "impossible."

The third case of an impossible problem, described a few boxes ago by the following quotation, is actually fairly rare.

> "You solve the problem completely without using one of the given facts. You check your solution against the facts that you actually used, and everything matches. However, your final Venn Diagram contradicts this one fact that you did not use. This contradiction means that your solution does not satisfy all the given facts, and furthermore, that no Venn Diagram could do so.

To see an example of this, imagine that we take the first version of the NYPD problem, and we add an additional constraint: "17 speak both French and Spanish." This would make the problem impossible, because the given constraints result in 15 officers speaking both Spanish and French. (You can verify this by looking at the answers on Page 146.) Whatever the true number happens to be, it cannot simultaneously be 15 and 17. Therefore, the problem cannot have a solution after adding this additional constraint of 17 officers speaking both French and Spanish.

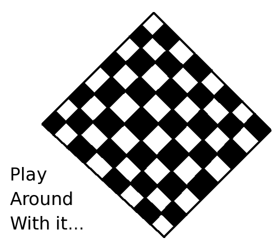


\# 1-4-11

We now cheerfully return to problems that are possible. In this problem, you are the training officer for a new 80-agent spy unit doing extremely secret activities in the Middle East.

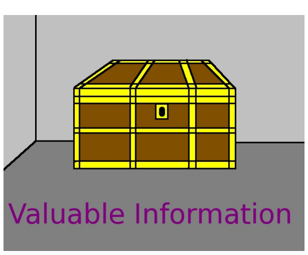
- The unit was built around an elite cadre of three experienced veterans who are certified in Arabic, Farsi, and parachuting.
- There are 30 members of the unit who are certified in Arabic.
- There are 29 members of the unit who are certified in Farsi.
- The number of members of the unit certified in parachuting is 34.
- The unit just recently received a new batch of 24 agents who have yet to be trained in any of these 3 vital skills.
- There are 6 members who are academic linguists hired from universities, who know both Farsi and Arabic but who cannot parachute (and probably will never learn how).
- From a previous mission you know that there are 10 members of the unit who know Arabic and who can parachute.

Using these facts, draw up a Venn Diagram that represents the skills mix for the unit as described here. For simplicity, put parachuting in the top circle; in the left circle put Arabic, and in the right circle put Farsi. The answer can be found on Page 147.



\# 1-4-12

Just for fun, redo the previous problem, but with the size of the unit being 90 agents. The answer can be found on Page 147.



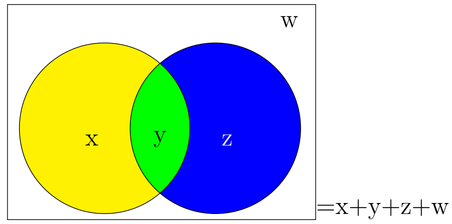
Using the techniques we learned earlier, namely putting an x in one of the regions of the Venn Diagram, you can solve the next problem. However, there is a short cut formula which is a huge time saver. It is very easy to understand as well.

$$\#(\mathcal{A} \cup \mathcal{B}) = \#\mathcal{A} + \#\mathcal{B} - \#(\mathcal{A} \cap \mathcal{B})$$

In a way, this formula allows you to change between $\cap$ and $\cup$ very freely. This formula is called "the inclusion-exclusion formula for sets" in this textbook. A less common name is to call it "the union-intersection formula." In words, we would say "The size of the union of the two sets is equal to the size of one, plus the size of the other, minus the size of the intersection."

This formula has a twin, called "the inclusion-exclusion formula for probability," given on Page 325. Together, these formulas are extremely useful.

Consider the following Venn Diagram for a situation where there are x objects in $\mathcal{A}$ that are not in $\mathcal{B}$, and likewise there are z objects in $\mathcal{B}$ that are not in $\mathcal{A}$. Also, there are y objects in $\mathcal{A} \cap \mathcal{B}$. Finally, while we'll have no use of the fact, there are w items in neither $\mathcal{A}$ nor $\mathcal{B}$.



Take a moment to verify that this is the correct Venn Diagram for this situation, and continue in the next box.

Now that you have verified the Venn Diagram in the previous box, observe that

$$\begin{aligned} \#\mathcal{A} &= x + y \\ \#\mathcal{B} &= y + z \end{aligned}$$

and adding those two equations together makes

$$\#\mathcal{A} + \#\mathcal{B} = x + 2y + z$$

If it comes to pass that we want to know the size of $\mathcal{A} \cup \mathcal{B}$, then $x + 2y + z$ isn't what we want. We want instead, $x + y + z$. Therefore, we should proceed as follows.

$$\begin{aligned} \#\mathcal{A} + \#\mathcal{B} &= x + 2y + z \\ \#\mathcal{A} + \#\mathcal{B} &= (x + y + z) + y \\ \#\mathcal{A} + \#\mathcal{B} &= \#(\mathcal{A} \cup \mathcal{B}) + y \\ \#\mathcal{A} + \#\mathcal{B} &= \#(\mathcal{A} \cup \mathcal{B}) + \#(\mathcal{A} \cap \mathcal{B}) \\ \#\mathcal{A} + \#\mathcal{B} - \#(\mathcal{A} \cap \mathcal{B}) &= \#(\mathcal{A} \cup \mathcal{B}) \end{aligned}$$

That's the derivation of this important formula. As you can see, it is just a matter of correcting the sum to avoid accidentally counting the y part twice.

The next example is one that we saw in the previous module, on Page 107. At that time, it was a rather long problem. However, we will be able to solve it much more quickly now, because we have learned the inclusion-exclusion formula:

$$\#(\mathcal{A} \cup \mathcal{B}) = \#\mathcal{A} + \#\mathcal{B} - \#(\mathcal{A} \cap \mathcal{B})$$

For Example :

1-4-13

Perhaps you are interning with a staffing firm that specializes in hiring programmers. A software company is looking to expand, and your firm is hired to help them find the necessary talent. The programmers must know the computer languages `C++` and `Python`. Your firm receives 87 applications. Luckily, 75 applications include knowledge of at least one of the languages. As it comes to pass, 48 applicants know `Python`, which is a good start, but 31 applicants do not know `C++`. How many people know both languages?

$$\begin{aligned}
\#\left(\mathcal{A}\cup\mathcal{B}\right) &= \#\mathcal{A}+\#\mathcal{B}-\#\left(\mathcal{A}\cap\mathcal{B}\right)\\
\#\left(\mathcal{P}\cup\mathcal{C}\right) &= \#\mathcal{P}+\#\mathcal{C}-\#\left(\mathcal{P}\cap\mathcal{C}\right)\\
75 &= 48+(87-31)-\#\left(\mathcal{P}\cap\mathcal{C}\right)\\
75-48-87+31 &= -\#\left(\mathcal{P}\cap\mathcal{C}\right)\\
-29 &= -\#\left(\mathcal{P}\cap\mathcal{C}\right)\\
29 &= \#\left(\mathcal{P}\cap\mathcal{C}\right)
\end{aligned}$$

Therefore, we can conclude that 29 people know both languages. By the way, in case you are curious about the "$(87-31)$" that appears as $\mathcal{C}$, note that there are 87 people in the survey, and 31 do not know `C++`. Therefore, $87-31=56$ must know `C++`.

Play Around With it...

1-4-14

In a survey of freshmen students, it is found that 857 need remedial math courses, and 196 need remedial English courses. There are 77 students who need both. If there are 2858 students in the freshman class,...

- How many students need either remedial English or math? [Answer: $857+196-77=976$.]
- How many students need neither remedial English nor math? [Answer: $2858-976=1882$.]

Play Around With it...

1-4-15

Likewise, we saw this question in the previous module, on Page 105. As before, we will be able to solve it much more quickly now that we have the inclusion-exclusion formula.

A restaurant is contemplating a liquor license and the owner's cousin is asked to survey people as they are waiting to be seated. He asks if they like beer or wine. The cousin does write down that 104 customers like beer and 67 customers like wine. However, he did not write down how many people like both. He surveyed 135 people. All is not lost, however, as he remembers that only two couples (4 people) said that they neither liked wine nor beer.

- How many people like either wine or beer? [Answer: 131.]
- Tell me how many people like both wine and beer? [Answer: 40.]

Play Around With it...

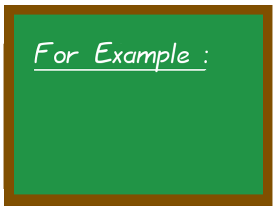
1-4-16

We saw this question earlier also, on Page 104. Once again, we will be able to solve it much more quickly now that we have the inclusion-exclusion formula.

Let us consider that a sports arena is surveying frequent visitors to see what type of concession stand to offer. They survey 408 people and it turns out that 198 would like fried chicken, and 205 would like health food. However, 106 want both options.

- How many people would be happy with either option? [Answer: 297.]
- How many people would be happy with neither option? [Answer: 111.]

The next problem appeared in *Finite Mathematics & Its Applications* by Larry J. Goldstein, David I. Schneider and Martha J. Siegel. It was in Chapter 5, Section 3, Example 2, of the Seventh Edition.



1-4-17

The presidents of the top 500 corporations in the USA were surveyed by the magazine *Executive* in the year 2000. There were 500 of them (of course) and 310 had business degrees—238 had undergraduate degrees, and 184 had graduate degrees. How many had exactly one degree in business? How many had both degrees in business? How many had neither? Did more presidents have or not have a Bachelor's Degree in business?

Let's denote the presidents with Bachelor's degrees by $\mathcal{B}$ and those with Master's degree's by $\mathcal{M}$. First, we can use the inclusion-exclusion formula:

$$\#(\mathcal{B} \cup \mathcal{M}) = \#\mathcal{B} + \#\mathcal{M} - \#(\mathcal{B} \cap \mathcal{M})$$

which becomes

$$310 = 238 + 184 - \#(\mathcal{B} \cap \mathcal{M})$$

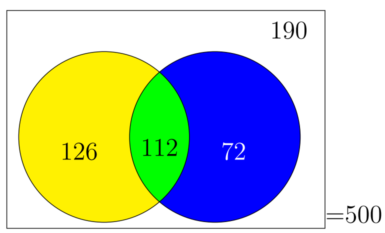
and therefore

$$\#(\mathcal{B} \cap \mathcal{M}) = 238 + 184 - 310 = 112$$

We will continue in the next box.

Continuing with the previous box, it is even easier is to find those corporate presidents with zero degrees in business. Since 310 have some sort of business degree out of 500, then surely 500-310=190 have no business degree at all. Next, we can fill in a Venn diagram. With 238 having undergraduate degrees, the left moon-shaped part should have $238 - 112 = 126$, while the right moon-shaped part should have $184 - 112 = 72$.

Then we have

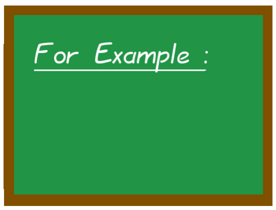


We can therefore say that 190 of them had zero degrees in business, and 112 had two degrees in business. Thus the number with one degree in business is $500 - 112 - 190 = 198$. Finally, while 238 have the Bachelor's in business, that means that $500 - 238 = 262$ do not have one. Thus the majority of Fortune-500 corporate presidents have their Bachelor's degree in something other than business.

Now we can easily check our work in the problem of the last three boxes.

- We are told 310 have some sort of business degree. $126 + 72 + 112 = 310$. Good.
- We are told 238 have undergraduate degrees. $112 + 126 = 238$. Good.
- We are told 184 have graduate degrees. $112 + 72 = 184$. Good.
- We are told 500 people are in the survey. $126 + 112 + 72 + 190 = 500$. Good.

It is very easy to make a silly mistake in these problems, and therefore you should definitely check your work.



\# 1-4-18

A vet is examining cats to figure out what is causing an outbreak of a mysterious feline illness. The technician surveyed the 87 customers that week. They were asked (1) if that cats were permitted outside, and (2) if the household also has a dog. There were 30 people who said “no” to both questions, and 20 answered “yes” to the outdoors question. While 47 answered “yes” to the dog question, the technician forgot to record how many answered “yes” to both questions. How many was that?

First, we mark outside the box that 87 customers were in the survey. Then 30 goes into the background area of the box, inside but not near or in any of the circles, to show that they answered no to both questions. Let dog-owners be the right circle, and people who let their cat outside be the left circle. So we know that the union of the two circles is $87 - 30 = 57$. Now we can use the inclusion-exclusion formula to get

$$\#(\mathcal{D} \cup \mathcal{O}) = \#(\mathcal{D}) + \#(\mathcal{O}) - \#(\mathcal{D} \cap \mathcal{O})$$

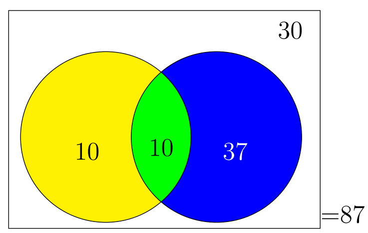
and then that is

$$57 = 47 + 20 - \#(\mathcal{D} \cap \mathcal{O})$$

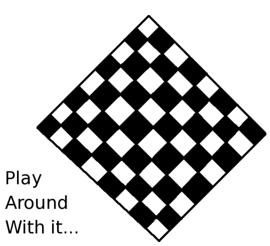
and as you can see $\#(\mathcal{D} \cap \mathcal{O}) = 10$. So we have 10 people who own dogs and let their cats out, 10 people let their cats out and don't own dogs, and finally 37 people own dogs but do not let their cats out.

The final Venn Diagram is in the next box.

Here is the Venn Diagram for the previous example.



First, I'd like you to try the next problem just as it is, without a hint. Then, read the hint in the next box. When you try the problem a second time, you'll find that it is tons easier.



\# 1-4-19

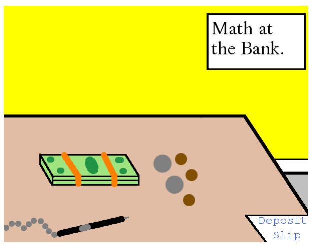
An election is being held to determine the chairperson of the board of directors of a newly founded charity. The candidates are Alice, Bob, and Charlene. Of the 70 members of the board, 8 would be satisfied with any of the three candidates, and 1 would not be satisfied with any of the candidates. There are 26 who would be satisfied with Alice, 39 who would be satisfied with Bob, and 47 who would be satisfied with Charlene. Moreover, when asked “Would you be satisfied with either Alice or Charlene?” the response was 56 votes; only 14 said neither would be satisfactory. Similarly, when asked “Would you be satisfied with either Bob or Charlene?” the response was 66 votes; only 4 said neither would be satisfactory. Finally, when asked “Would you be satisfied with either Alice or Bob?” the response was 51 votes; only 19 said neither would be satisfactory.

Draw a Venn Diagram to describe this situation. Let those who would be satisfied with Charlene be in the top circle. Let those who would be satisfied with Bob be in the lower-left circle, and those who would be satisfied with Alice be in the lower-right circle. Furthermore, tell me how many members of the board can be satisfied with more than one of the candidates.

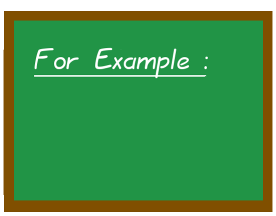
The solution can be found on Page 147.

Here's the hint for the previous problem. We are told "Would you be satisfied with Alice or Bob?" had 51 votes, and we know how many people would be satisfied with Alice, and how many would be satisfied with Bob. Use the 2-variable inclusion-exclusion formula to convert this into knowing how many people would simultaneously be satisfied by Alice AND be satisfied by Bob. Similarly, use inclusion-exclusion formula to find the "ANDs" for Bob and Charlene. Finally, use that formula to find how many people would simultaneously be satisfied by Alice AND be satisfied by Charlene.

Armed with these intersections, the problem completely becomes trivial after that.



Coming soon: a description of what a hedge fund is. That isn't ready yet. Sorry.



1-4-20

Let us imagine that you and Jed are interning in the offices of an elite hedge fund. Rather than hire 2–3 in-house economists to decide where to invest, this fund has decided to do a massive survey of 150 academic economists, and ask them which of three industries (biomedical, software, or telecommunications) is going to be a profitable focus for the fund in the coming year. Jed and yourself were tasked with surveying the economists, who have been flown to a big event in Manhattan hosted by the fund. After you both carefully collect the results, Jed accidentally spills coffee on the data. Luckily, only one datum is obfuscated, marked "(stain)" below. Your task is to summarize the data with a Venn Diagram. Will you be able to do this, despite the loss of one data point?

The data is in the next box.

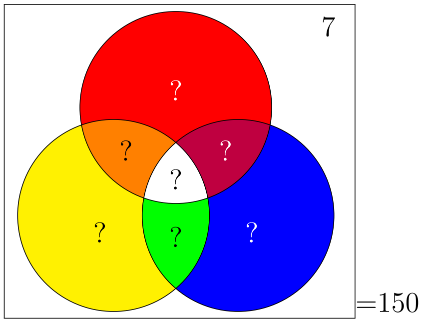
Here is the data for our present example, which we started in the previous box.

- 7 economists gave a positive outlook to none of those three industries.
- 113 economists gave a positive outlook to the software industry.
- 99 economists gave a positive outlook to the biomedical industry.
- 98 economists gave a positive outlook to the telecommunications industry.
- 79 economists gave a positive outlook to both software & biomedical.
- 84 economists gave a positive outlook to both software & telecommunications.
- 82 economists gave a positive outlook to both telecommunications & biomedical.
- (stain) gave a positive outlook to all three industries.

We will proceed to solve the problem in the next few boxes.

When first glancing at the data in the previous box, we can put the 150 with the equal sign outside of the diagram, and we can put the 7 in the background of the diagram. That's nice, as we have 2 of the 9 spots already given to us.

Next, we might be a bit dismayed to discover that all the remaining data points cover more than one region of the Venn Diagram. This gives us the rather dismal starting point:



By the way, I'm using the convention that the software industry is the lower-right circle, the biomedical industry is the lower-left circle, and the telecommunications industry is the top circle. We will continue in the next box.

Continuing with the previous box, let's see what happens if we put an x in the center of the Venn Diagram.

1. Because 79 economists gave a positive outlook to both software & biomedical, and we have x economists for all three, then $79 - x$ goes in the mitre-shaped region between the software circle and the biomedical circle.

2. Because 84 economists gave a positive outlook to both software & telecommunications, and we have x economists for all three, then $84 - x$ goes in the mitre-shaped region between the software circle and the telecommunications circle.

3. Because 82 economists gave a positive outlook to both biomedical & telecommunications, and we have x economists for all three, then $82 - x$ goes in the mitre-shaped region between the biomedical circle and the telecommunications circle.

4. Now we have to decide what to put in the moon-shaped region in the software circle, in order to make the whole circle 113. We've already accounted for x, $79 - x$, and $84 - x$. We should write
$$113 - x - (79 - x) - (84 - x)$$
which simplifies to
$$113 - x - 79 + x - 84 + x$$
or more simply $x - 50$, since $113 - 79 - 84 = -50$.

5. Next, we have to decide what to put in the moon-shaped region in the biomedical circle, in order to make the whole circle 99. We've already noted $79 - x$, x, and $82 - x$. We should write
$$99 - (79 - x) - x - (82 - x)$$
which simplifies to
$$99 - 79 + x - x - 82 + x$$
or more simply $x - 62$. Again, note that $99 - 79 - 82 = -62$.

We will continue in the next box.

Continuing with the previous box,

6. Similarly, we have to decide what to put in the moon-shaped region of the telecommunications circle, in order to make the entire circle add to 98. We have so far $82 - x$, $84 - x$, and x. We need to put

$$98 - (82 - x) - (84 - x) - x$$

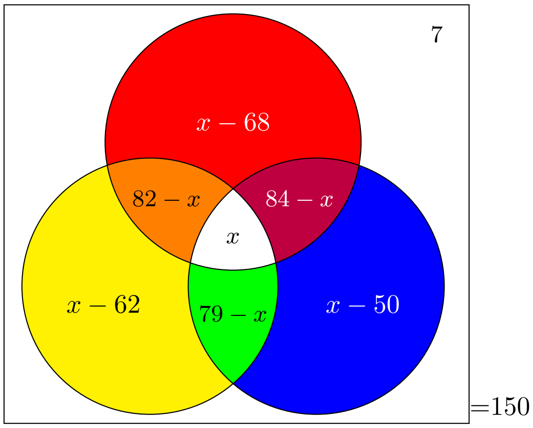
which simplifies to

$$98 - 82 + x - 84 + x - x$$

or more compactly $x - 68$.

If you've been following the steps in the previous two boxes carefully, then your Venn Diagram should look like the one in the next box.

Continuing with the last few boxes, at this stage, we have the following diagram:



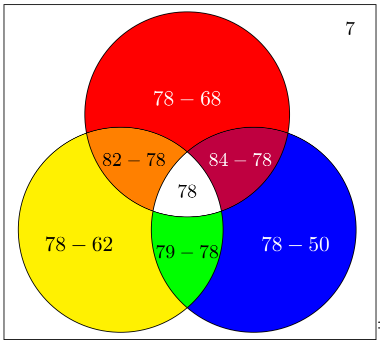
The previous box is perhaps one of the most intimidating Venn Diagrams in the whole of set theory. Unfortunately, this problem is about to get even messier before we're finished with it.

At this point, we should realize that all eight internal regions of the Venn Diagram must add up to 150. That means we have

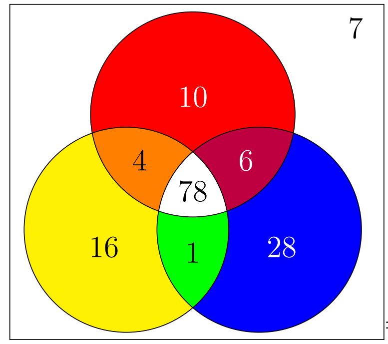
$$\begin{aligned}(x-68)+(84-x)+(x-50)+(79-x)+(x-62)+(82-x)+x+7 &= 150\\ (-68)+84+(-50)+79+(-62)+82+7+x-x+x-x+x-x+x &= 150\\ 72+x &= 150\\ x &= 150-72\\ x &= 78\end{aligned}$$

We can now finish the problem in the next box.

We can plug $x = 78$ into the algebraic expressions in the Venn Diagram to get



=150 which simplifies to



=150

Perhaps now, we should check our work.

We will now check our work for the problem that spanned the previous nine boxes.

- We were told 7 gave a positive outlook to none of these three industries. That matches our diagram.
- We were told 113 gave a positive outlook to the software industry. $28 + 1 + 78 + 6 = 113$. Good.
- We were told 99 gave a positive outlook to the biomedical industry. $16 + 4 + 78 + 1 = 99$. Good.
- We were told 98 gave a positive outlook to the telecommunications industry. $10+4+78+6 = 98$. Good.
- We were told 79 gave a positive outlook to both software & biomedical. $78 + 1 = 79$. Good.
- We were told 84 gave a positive outlook to both software & telecommunications. $6 + 78 = 84$. Good.
- We were told 82 gave a positive outlook to both biomedical & telecommunications. $78+4 = 82$. Good.
- Oh, and now we know that the coffee stain obliterated "78 gave a positive outlook to all three industries."
- Last but not least, we were told there were 150 economists in the survey:

$$28 + 1 + 16 + 6 + 78 + 4 + 10 + 7 = 150 \leftarrow \text{ Good!}$$

Since all these match, we can be confident that we have done the problem correctly.

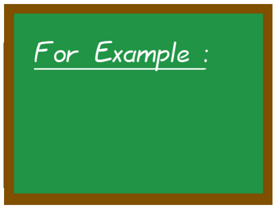
Earlier, we saw the inclusion-exclusion formula for (two) sets:

$$\#(\mathcal{A} \cup \mathcal{B}) = \#\mathcal{A} + \#\mathcal{B} - \#(\mathcal{A} \cap \mathcal{B})$$

However, there is another version, for three sets:

$$\#(\mathcal{A} \cup \mathcal{B} \cup \mathcal{C}) = \#\mathcal{A} + \#\mathcal{B} + \#\mathcal{C} - \#(\mathcal{A} \cap \mathcal{B}) - \#(\mathcal{B} \cap \mathcal{C}) - \#(\mathcal{A} \cap \mathcal{C}) + \#(\mathcal{A} \cap \mathcal{B} \cap \mathcal{C})$$

While that formula is a little bit large and complicated, we'll see shortly how it can be used to make quick work of our previous example.



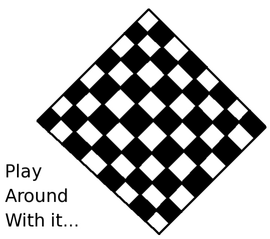
1-4-21

Let's re-examine the previous example, using our new tool.

Let $\mathcal{A}$ be the set of economists who give a positive outlook to the software industry. Likewise, let $\mathcal{B}$ and $\mathcal{C}$ represent the biomedical and telecommunications industries. Using the three-set version of the inclusion-exclusion formula, we obtain

$$\begin{aligned}
\#(\mathcal{A} \cup \mathcal{B} \cup \mathcal{C}) &= \#\mathcal{A} + \#\mathcal{B} + \#\mathcal{C} - \#(\mathcal{A} \cap \mathcal{B}) - \#(\mathcal{B} \cap \mathcal{C}) - \#(\mathcal{A} \cap \mathcal{C}) + \#(\mathcal{A} \cap \mathcal{B} \cap \mathcal{C}) \\
150 - 7 &= 113 + 99 + 98 - 79 - 82 - 84 + x \\
143 &= 65 + x \\
143 - 65 &= x \\
78 &= x
\end{aligned}$$

Once we have x, the rest of the problem is easy. We have now mathematically undone the coffee stain. Not only that, it required a lot less work than finding x via "the long way," as we did in the preceding pages.



1-4-22

Having "undone" Jed's coffee stain in the previous example, we now have the following facts:

- 7 economists gave a positive outlook to none of those three industries.
- 113 economists gave a positive outlook to the software industry.
- 99 economists gave a positive outlook to the biomedical industry.
- 98 economists gave a positive outlook to the telecommunications industry.
- 79 economists gave a positive outlook to both software & biomedical.
- 84 economists gave a positive outlook to both software & telecommunications.
- 82 economists gave a positive outlook to both telecommunications & biomedical.
- 78 economists gave a positive outlook to all three industries.

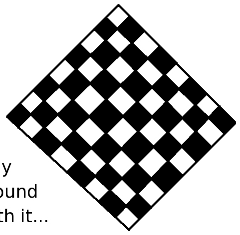
From this starting point, draw a complete Venn Diagram that describes the entire situation. Compare your answer with the solution given on Page 135.

It might be worthwhile to verify the three-set inclusion-exclusion formula on the three-circle Venn Diagram problem about the mushrooms. The final diagram was given on Page 122. We had tasty mushrooms, which we can denote $\mathcal{T}$; we had spotted mushrooms, which we can denote $\mathcal{S}$; lastly, we had lethal mushrooms, which we can denote $\mathcal{L}$.

$$\begin{aligned}
\#(\mathcal{L} \cup \mathcal{T} \cup \mathcal{S}) &= \#\mathcal{L} + \#\mathcal{T} + \#\mathcal{S} - \#(\mathcal{L} \cap \mathcal{T}) - \#(\mathcal{L} \cap \mathcal{S}) - \#(\mathcal{T} \cap \mathcal{S}) + \#(\mathcal{L} \cap \mathcal{T} \cap \mathcal{S}) \\
(995 - 29) &= 611 + 345 + 263 - 97 - 114 - 48 + 6 \\
966 &= 966
\end{aligned}$$

Great! It works.

The following fairly difficult problem appeared in *Finite Mathematics and Calculus with Applications* by Margaret Lial, Raymond Greenwell, and Nathan Ritchey. It was Problem 61 of Chapter 7, Section 2, of the 9th Edition. Let's see if you can do it.



Play
Around
With it...

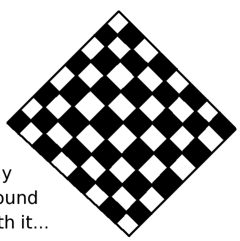
1-4-23

A chicken farmer surveyed his flock with the following results. The farmer had

- 9 fat red roosters,
- 13 thin brown hens,
- 15 red roosters,
- 11 thin red chickens,
- 17 red hens,
- 56 fat chickens,
- 41 roosters,
- and 48 hens.

Assume that all chickens are either thin or fat, are either red or brown, and are either hens or roosters. Make a Venn Diagram to describe this situation. (Hint: sometimes there are students who think that this cannot be done with a 3-circle Venn Diagram—but it definitely can be done.) For my readers who are not native speakers of English, I should add the following additional hint: a female chicken is called a hen, while a male chicken is called a rooster.

The answers can be found on Page 148.



Play
Around
With it...

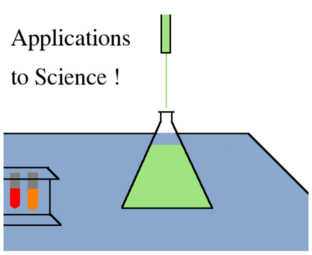
1-4-24

Continuing with the previous box, answer the following ancillary questions:

- How many chickens are in the flock?
- How many chickens are red?
- How many fat roosters are there?
- How many fat hens are there?
- How many thin brown chickens are there?
- How many red fat chickens are there?

As noted, the answers can be found on Page 148.

Applications
to Science !



One of the super-classic applications of set theory is the typing system used for human blood. Probably most of my readers have heard of O+, B-, AB+, et cetera. Using set theory, you will see that these concepts can be explained far easily than you might at first suspect. Moreover, we're going to go into much more detail than most texts.

For example, given the blood type of a person, you can very easily determine the blood types that this person can receive, and the blood types of patients that this person can safely donate blood to. It's just the concept of a subset or a superset—nothing more.

Hard but Valuable!

As I mentioned in the previous box, we're going to talk about blood types now. Some instructors will cover the material, on blood types, that follows, but other instructors will not. Of course, if your instructor is covering this material, then you should definitely read the rest of this module. However, if your instructor is not covering this material, then you should probably read the rest of this module anyway.

Not only is being knowledgeable on the basics of medicine an important part of being a well-educated person, this particular information is very useful in preparing you to either care for your parents as they age, or to care for your children if you happen to ever have any. Also, blood types are a super-classic and famous use of Venn Diagrams, found in nearly every textbook of this subject.

After all, you've done the hard work of learning set theory—why not do a tiny amount of extra work and learn about how set theory is used to save human lives, every day.

Historical Note:

The idea of blood transfusion is actually quite old. There is one late medieval/early renaissance case, namely Pope Innocent VIII (1432–1492) on his deathbed. Most of the early experiments with blood were performed in the 1600s, but usually the patient did not survive. A pioneer of the subject was Jean-Baptiste Denys (1643–1704), physician to Louis XIV (1638–1715).

Several medical procedures in that era had roughly 80% death rates, but they were used often if the death rate after doing nothing would have been 100%.

At some point, it was noticed that if the blood donor were a close relative of the patient, then the patient was much more likely to survive. This tantalizing clue might have eventually lead to the genetic theory of blood types. Unfortunately, toward the end of the 1600s, the procedure was banned in Europe for religious reasons.

Historical Note:

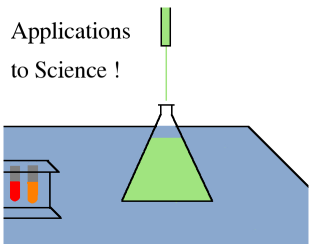
The modern procedure was pioneered by an English physician Dr. James Blundell (1791–1878). One of his techniques was to mix a bit of blood from the donor and the patient and see (visually) if it formed clumps—which we now know as blood clots. If it did, then he would not transfuse, and this made the procedure far safer. Nonetheless, sometimes the blood did not clot in the test, and the patient died anyway. In either case, the causes of the compatibility, or incompatibility, were not known.

Finally, in 1901 Karl Landsteiner (1868–1943), an Austrian physician who spent much of his career in the USA, discovered blood types. At last, there was an explanation!

Historical Note:

The pioneer of blood typing was Karl Landsteiner (1868–1943), and he won the Nobel Prize in Physiology & Medicine in 1930. He took his medical degrees from the University of Vienna, becoming a doctor of medicine in 1891. He studied chemistry next for two years, in Würzburg, then returned to Vienna. His research prospered until The First World War 1914–1919, where he served in a military hospital. After the war, Austria was financially devastated, and he was unable to continue with his research. He moved to the Hague in the Netherlands, and then to the Rockefeller Institute in New York City, where he lived for the rest of his life.

He discovered the A-B-AB-O system first, in the year 1901, and then in 1909 discovered how polio was spread and later made the first vaccine for it. Polio was an extremely debilitating disease, crippling many of the world's population, including Franklin Delano Roosevelt (1882–1945). The Rh-factor was an important extension of the A-B-AB-O scheme, and Landsteiner was a co-discoverer of that as well, in 1937. He died in 1943 while working in his laboratory. He was 75 years old at the time, and it is said that he never stopped researching.



There are receptors on the surface of red blood cells, called antigens. Two particular receptors are very important, and are called "A" and "B."

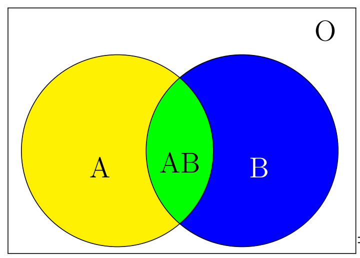
- If you've got the "A" antigen, but not "B," we say you have blood type "A."
- If you've got the "B" antigen, but not "A," we say you have blood type "B."
- If you've got both, then we say you've got blood type "AB."
- Finally, if you have neither, we say you have blood type "O."

At this moment, in the language of set theory, we know about the antigens $\{A, B\}$, and the four subsets of this set, namely,

$$\{\{\}\,; \{A\}\,; \{B\}\,; \{A, B\}\}$$

correspond to four possible blood types O, A, B, and AB respectively. (By the way, in Europe the O is usually written as zero.)

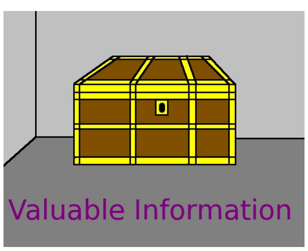
This arrangement can be summarized with the following diagram:



=Everybody

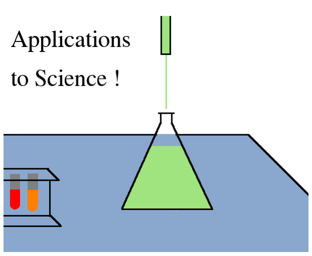
where the left circle represents people with the A antigen, and the right represents people with the B antigen. Alternatively, here is another way to visualize the relationship.

Sets of Antigens:	$\{\}$	$\{A\}$	$\{B\}$	$\{A, B\}$
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
Blood Types:	O	A	B	AB



- When the patient receives blood, if the donor has an antigen which the patient does not, then there will be a major problem.
- Thus, the set of antigens of the donor must be a subset of the antigens of the patient.

One way to remember this is that no new antigens may be introduced when giving a patient blood.



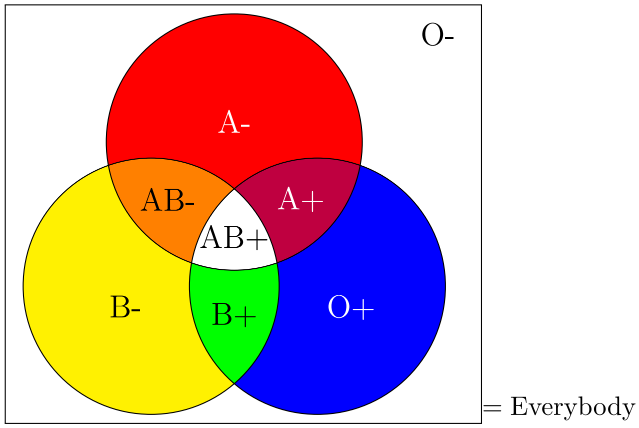
As often happens in science, the original theory was in need of improvement. About 30 years later, another antigen was discovered. This one is called Rh-factor.

Either you have the Rh factor, or you don't. If you have it, we affix a plus sign to the end of your blood type. If you do not have it, then we affix a minus sign to the end of your blood type. However, the same rules apply as given in the previous box. Now, we know about a set of three antigens, namely $\{A, B, R\}$, and a blood type is one of the 8 possible subsets of this set. The following chart explains the relationship.

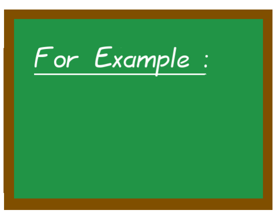
Sets of Antigens:	$\{\}$	$\{A\}$	$\{B\}$	$\{A, B\}$	$\{R\}$	$\{A, R\}$	$\{B, R\}$	$\{A, B, R\}$
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
Blood Types:	O-	A-	B-	AB-	O+	A+	B+	AB+

Take a moment to understand this relationship.

The Venn Diagram now looks like this:



where the top circle represents those with the A antigen, and the lower-left circle is those with the B antigen. Likewise the lower-right circle are those with the Rh antigen.

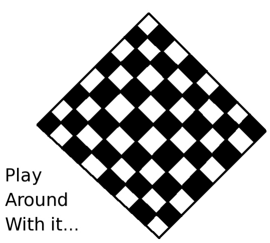


1-4-25

Suppose Alice and Bob are on vacation, and they get into a bad car accident, and require blood transfusions. Alice has blood type AB- and Bob has blood type B+. What types of blood can Alice get? What types of blood can Bob get?

Since Alice has blood type AB-, then the antigens she has are $\{A, B\}$, and she can receive any subset of those, which would be $\{\}$, $\{A\}$, $\{B\}$, or $\{A, B\}$. Note, since the antigen set has 2 elements, it must have $2^2 = 4$ subsets (see Page 51). A medical doctor would call these blood types O-, A-, B-, and AB-.

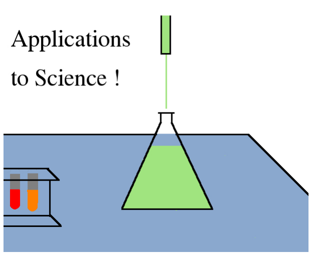
Since Bob has blood type B+, then the antigens he has are $\{B, Rh\}$, and he can receive any subset of that. Again, there are 2 things in the set, so the set has 4 subsets. Those subsets are $\{\}$, $\{B\}$, $\{Rh\}$, and $\{B, Rh\}$. Then a doctor would call those O-, B-, O+, B+.



1-4-26

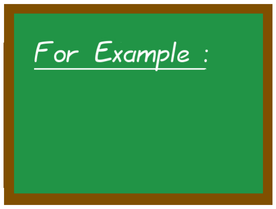
- Charlie has blood type A+. What types can he receive?
- David has blood type AB+. What types can he receive?
- Electra has blood type O+. What types of blood can she receive?

[Answer: Electra can receive either O- or O+. David can receive any of the eight types. Charlie can receive O+, O-, A+, and A-.]



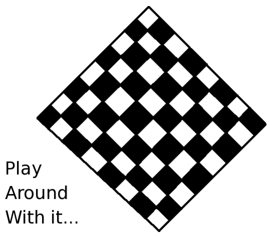
We saw in the above box that David has all three antigens, therefore any set of antigens is a subset of his antigens. For this reason, AB+ is called "the universal recipient."

On the other hand, do you remember (from Page 50) that we said that the empty set is a subset of any set? This means that someone with the empty set as their antigens can donate blood to anyone. For this reason, O- is called "the universal donor."



1-4-27

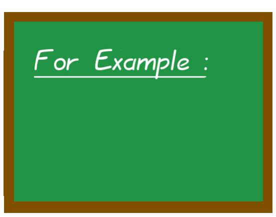
- Who can receive Alice's blood? Recall, she is of type AB-.
- Since her antigens are $\{A, B\}$, then the supersets of that set are $\{A, B, Rh\}$ and $\{A, B\}$, so only patients of type AB+ and AB- can receive her blood.
- Who can receive Bob's blood? Recall, he is of type B+.
- Since his antigens are $\{B, Rh\}$ then the supersets are $\{B, Rh\}$ and $\{A, B, Rh\}$. This means he can donate to people of blood types B+ and AB+.



1-4-28

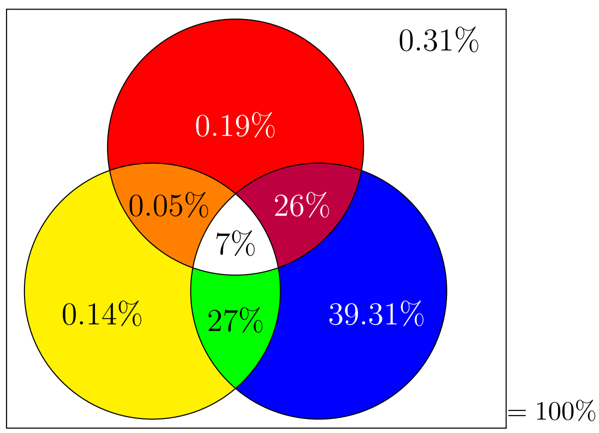
- What blood types of patients can receive Charlie's blood? (He's A+)
- What blood types of patients can receive David's blood? (He's AB+)
- What blood types of patients can receive Electra's blood? (She's O+)

[Answer: David can give to only AB+ patients. Electra can give to O+, A+, B+, and AB+. Charlie can give to only to those with types A+ and AB+.]



1-4-29

Let's imagine Bob goes to Hong Kong. In Hong Kong, the distribution of blood types is as follows:



and in this diagram, as before, the A antigen is the circle on the top, the B antigen is the circle on the left, and the Rh antigen is the circle on the right. (The data is taken from the Wikipedia article *Blood Type*, accessed on August 19th, 2010.) How many people can donate to someone who has type B+?

Type B+ participates in the B circle (left) and the Rh circle (right). So only the top circle is a problem. The top circle has

$$0.19\% + 0.05\% + 7\% + 26\% = 33.24\%$$

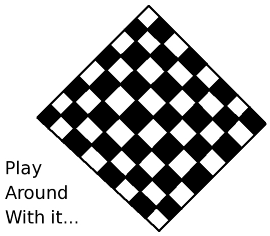
and those are the people whose blood would be toxic to Bob. Thus $100 - 33.24\% = 66.76\%$ of the people are safe.

An alternative approach to the above problem would be to realize that Bob can receive blood from someone of type B+, B-, O+, or O-. That means

$$0.14\% + 27\% + 39.31\% + 0.31\% = 66.76\%$$

of the people are safe.

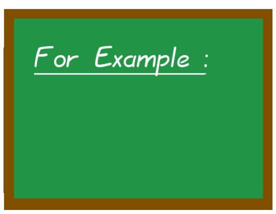
Of course, you are welcome to calculate it both ways, as a check on your work.



1-4-30

Remaining in Hong Kong, and using the Venn Diagram from the previous section,

- How many people can donate to someone who has type AB-?
  [Answer: 0.69% of the population.]
- How many people can donate to someone who has type O+?
  [Answer: 39.62% of the population.]

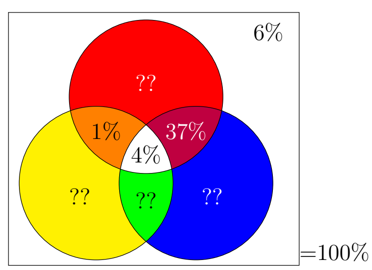


1-4-31

Suppose in a certain population, the A antigen is present in 49% of the population. Meanwhile, the B antigen is present in 15% of the population. On the other hand, the Rh antigen is present in 84% of the population.

Meanwhile, you also know that type O- is present in 6% of the population and AB+ is 4% of the population. Also AB- is 1% and A+ is 37%. Find the remaining percentages for the other 4 blood types.

- Let's start with a 3 circle Venn Diagram, with A on the top, B on the left, and Rh on the right, as we've been using from the beginning.
- We should immediately write in the size of the four given blood types.



We will continue in the next box.

Continuing with the previous box...

- At this time, you might have noticed the following fact: We have labelled all of the overlap regions except the one between the B and Rh groups, which represents B+. Let's put an x there.
- Then we know the top circle has 49%, so the A- part has to have 49%-1%-4%-37%=7% of the population.
- Likewise, the left circle has 15% of the population, so the B- part has to have 15%-1%-4%-x=10%-x.
- Similarly, the right circle has 84% of the population, so we know that it should have 84%-37%-4%-x%=43%-x

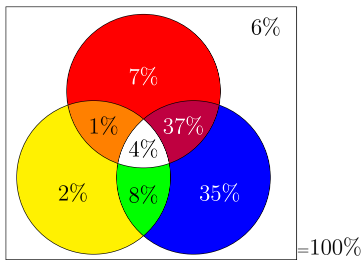
Continuing with the previous box, we also know that the entire population is included in the pie chart, so it must add to 100%. Let's try that

$$6\% + \underbrace{7\% + (10\% - x) + (43\% - x)} + \underbrace{1\% + 37\% + x} + \underbrace{4\%} = 108\% - x = 100\%$$

so then clearly $x = 8\%$.

Naturally the B- part is 10%-8%=2% and the O+ part is 43%-8%=35%.

Here is the final Venn Diagram for the example of the last several boxes.



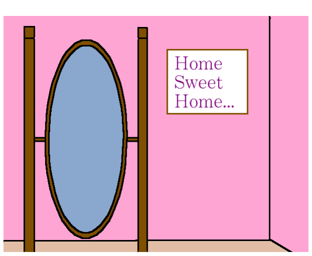
If you are curious, this is the distribution of blood types in Denmark.

We should check that the large circles, and the entire diagram, add to the correct values:

$$\begin{aligned} 7\% + 2\% + 1\% + 35\% + 37\% + 8\% + 4\% + 6\% &= 100\% \\ 7\% + 1\% + 4\% + 37\% &= 49\% = \text{A antigen} \\ 2\% + 1\% + 4\% + 8\% &= 15\% = \text{B antigen} \\ 35\% + 37\% + 4\% + 8\% &= 84\% = \text{Rh antigen} \end{aligned}$$

In case you are curious, while typesetting this box I realized there was an error in the previous example (which I have now fixed). If I hadn't checked my work, then I wouldn't have caught the error.

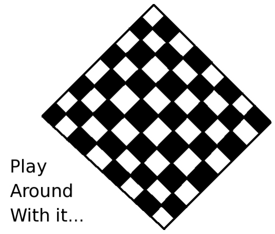
It is very important to check your work!



A Pause for Reflection...

The problems about blood types might seem a bit grim, or perhaps gross if you are bothered by blood. It's important to know about these things, in case someone near to you has a medical emergency, especially overseas. Certainly, the blood-type problems aren't as fun as the ice-cream flavor problem on Page 92 or the pizza-topping selection problem on Page 36 or Page 122.

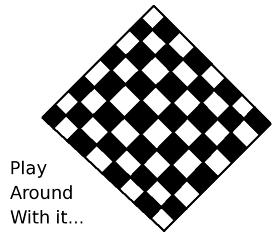
Well, if you like, you can think of the problems about blood types as analogous to a pizza-topping selection problem, but for vampires.



1-4-32

Suppose that in a particular country the A antigen is present within 29.5% of the population, and the B antigen is present in 38.6% of the population. Likewise, the Rh factor is present in 95.9% of the population. The type O- is 2%, and the type AB+ is 6.4%. Also, A+ is 22.1% of the population and B+ is 30.9% of the population. What percentage of the population is each blood type?

[Answer: O+= 36.5%, A+= 22.1%, B+= 30.9%, AB+= 6.4%, O-= 2%, A-= 0.8%, B-= 1.1%, AB-= 0.2%.] (This happens to be India.)

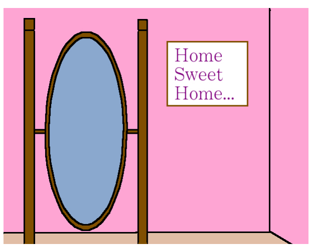


1-4-33

Suppose that in a particular country the A antigen is present within 45% of the population, and the B antigen is present in 13% of the population. Likewise, the Rh factor is present in 80.5% of the population. The type O- is 9%, and the type A+ is 34%. Also, B+ is 8%, and AB- is 0.5%. What percentage of the population is each blood type?

Hint: If you get stuck or frustrated, then you can think of this problem as similar to the pizza-topping selection example from Page 122, but for vampires. (Different vampires might prefer different blood types, for flavor.) It might help to go back and review that example.

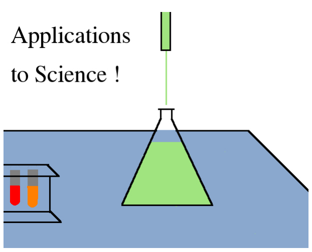
[Answer: O+= 36%, A+= 34%, B+= 8%, AB+= 2.5%, O-= 9%, A-= 8%, B-= 2%, AB-= 0.5%.] (This happens to be Brazil.)



A Pause for Reflection...

Why is it that the answers to the questions "who can receive so-and-so's blood" and "who can donate to so-and-so" always have either 1, 2, 4, or 8 answers?

Here is a hint: it has to do with the number of possible subsets of a set.



At this point, our exploration of blood types comes to a close. We'll revisit the topic later. Meanwhile, you might be interested if these eight blood types are the whole story, but the answer is "absolutely not!" There are many other antigens. These eight are the large-scale categories.

Modern blood research has lists of over 50 antigens that are checked. Most of these are relatively rare or have minimal clinical impact. However, some types, like the Duffy-antigen (D), affect the ability of certain people to travel to certain places. Interestingly enough, blood type affects organ transplants as well. Furthermore, those who get a bone-marrow transplant will often have their blood type change after the transplant. After all, the bone-marrow is where blood is manufactured.

You have now completed this module. All that remains is a listing of the answers to a few checkerboards from earlier in the module.

These are the tables for the checkerboard box on Page 118, where you were asked to convert some Venn Diagrams to tables.

	MS-Word		
MS-Excel	Has It	Needs It	Total
Has it	77	6	83
Needs it	12	5	17
Total	89	11	100

This is the solution to the other Venn-to-Table conversion.

	Languages Spoken		
Study Abroad	2 Langs.	3+ Langs.	Total
Did It	108	190	298
Didn't	84	96	180
Total	192	286	478

This is the solution to the two ice-cream table problems from Page 119.

	Chocolate		
Vanilla	Like	Dislike	Total
Like	40	10	50
Dislike	20	30	50
Total	60	40	100

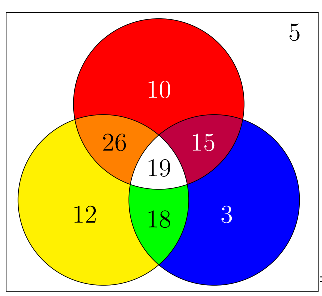
Before the Error

	Chocolate		
Vanilla	Like	Dislike	Total
Like	10	40	50
Dislike	50	0	50
Total	60	40	100

After the Error

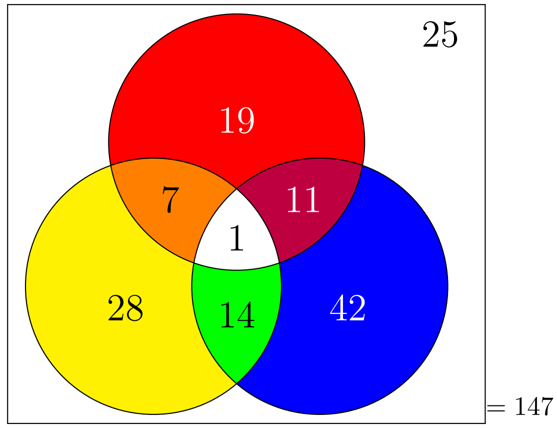
On Page 124, you were asked a question about the CS-recruiting business, with the twin of the evil intern from the pizza-topping selection problem. It turns out that there are 25 candidates who know only one language. That breaks down into 10 who know only Python, 12 who know only Java, and 3 who know only SQL.

Below is my Venn Diagram for the entire candidate pool. Recall that I said to put Python on top, Java on the left, and SQL on the right.



=108

The solution to Version 1 of the NYPD problem (from Page 125) will go here.



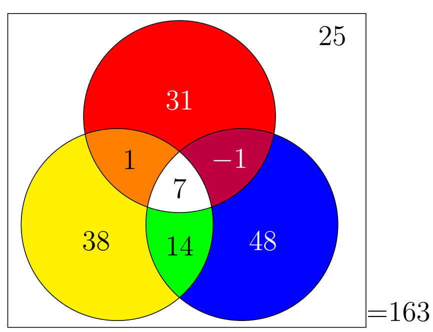
The top circle in my diagram represents Mandarin. The left circle represents French, and the right circle represents Spanish. You might have the circles rearranged, but the numbers should be the same.

Let's not forget the supplemental questions! There are 25 officers who speak none of these languages; there are 89 officers who speak exactly 1 of these languages; there are 32 officers who speak exactly two of these languages; there is 1 officer who speaks all three of these languages.

By the way, on tests, an amazingly large portion of the students forget to answer the supplemental questions. Often, after checking their Venn Diagram and seeing that it works, they joyously move on to other problems, leaving the supplemental questions blank.

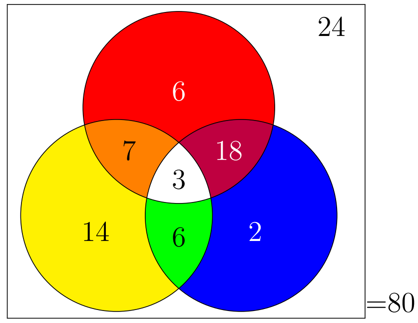
You should always make sure to answer all the questions that were asked.

The solutions to Version 2 of the NYPD problem (from Page 146) will go here.

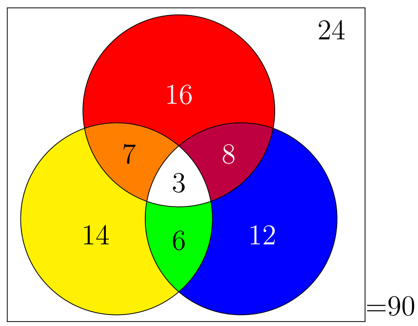


As you can see, there is a -1 in one of the spots. The size of a set can never be -1, or any negative number. For this reason, we know that something has gone horribly wrong. Usually, it is an arithmetic error, but in this case, the problem is broken.

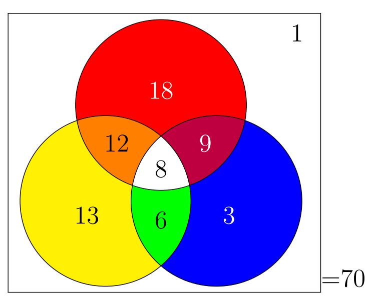
Here are the solutions to the first problem about an elite military unit (from Page 127).



Here are the solutions to the second problem about an elite military unit (from Page 127).



Here is the solution to the problem about electing Alice, Bob, or Charlene to the board of directors, as given on Page 131.

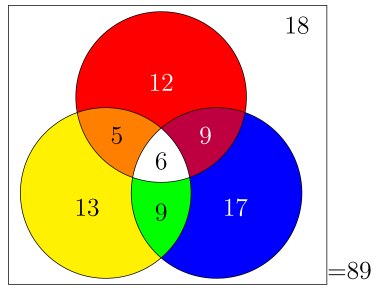


Remember, those who would be satisfied with Charlene are in the top circle. Those who would be satisfied with Bob are in the lower-left circle, and those who would be satisfied with Alice are in the lower-right circle. Finally, 35 members of the board are satisfiable by more than one candidate.

Here are the solutions to the problem about roosters, hens, and chickens (from Page 137).

For my Venn Diagram, I decided to let the top circle be red chickens, the lower-left circle be thin chickens, and the lower-right circle be male chickens (roosters). Therefore, brown chickens are outside the top circle, fat chickens are outside the lower-left circle, and female chickens (hens) are outside the lower-right circle.

However, there are many possible choices of how to organize your Venn Diagram. If you organized it differently, you should see the same numerals in different locations. The ancillary questions, given in the next box, should give you final confirmation that your Venn Diagram is correct for your organizational strategy.



Here are the answers for the ancillary questions for the problem about roosters, hens, and chickens (from Page 137).

- How many chickens are in the flock? [Answer: 89.]
- How many chickens are red? [Answer: 32.]
- How many fat roosters are there? [Answer: 26.]
- How many fat hens are there? [Answer: 30.]
- How many thin brown chickens are there? [Answer: 22.]
- How many red fat chickens are there? [Answer: 21.]