The Logic Game: Ten Levels of Problems Toward Mathematical Logic, and Set Theory

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Overview

In this assignment, we will explore the relationship between implications from mathematical logic, set theory, and the `if-then` notation in computer science. It is an exciting overlap, and it is actually a tiny taste of a very advanced subject called "Category Theory." (Just how learning about the slope of a line in middle-school algebra class is a foretaste of calculus.)

This assignment is divided into levels, like a video game. The levels are numbered 0, 1, 2, 3, . . . , 9. The first few levels are very easy, but Level 9 is fairly difficult. The idea is that by climbing up slowly through the levels, you will get skilled in a relatively smooth and mostly painless manner.

Most students will need to do about half of the problems in each level before moving on to the next. All the answers are posted to

`www.discrete-math-hub.com`

and this document can be found there too. You can decide for yourself when you are ready to move on from one level to the next.

I recommend the following schedule:

First Sitting: Levels Zero and One

Second Sitting: Levels Two and Three

Third Sitting: Levels Four and Five

Fourth Sitting: Level Six and Seven

Fifth Sitting: Level Eight

Sixth Sitting: Level Nine

Note: These questions make very easy-to-grade test questions. With my own students, I always put a few of these on the first test after teaching this material.

Example One:

Let's suppose that you get a co-op with a bio-informatics company, and you're debugging some badly written code about RNA. Each of the following sentences is equivalent.

- If "a sequence starts with ACG," then "that sequence starts with AC."
- "A sequence starting with AC" is necessary for "that sequence starting with ACG."
- "A sequence not starting with AC" implies "that sequence not starting with ACG."
- All "sequences that start with ACG" are "sequences that start with AC."
- The set of "sequences that start with ACG," is a subset of "the set of sequences that start with AC."

Yet, keep in mind, there are situations where a sequences starts with AC, but does not start with ACG. Consider ACUAGUCA, as an example.

Example Two:

Let's suppose that you get a co-op with a company that writes software for airline frequent-flyer programs. Each of the following sentences is equivalent.

- If "you flew more than 25,000 miles last year," then "you have gold status this year."
- The set of "the people who have gold status this year" is a superset of "the people who flew more than 25,000 miles last year."
- "Having flown more than 25,000 miles last year" requires "being given gold status this year."

- "Flying more than 25,000 miles last year," is sufficient for "being gold status this year."
- Not "being given gold status this year" requires not "having flown more than 25,000 miles last year."
- If "you do not have gold status this year," then "you did not fly more than 25,000 miles last year."
- "You have gold status this year," whenever "you flew more than 25,000 miles last year."
- The set of "the people who flew more than 25,000 miles last year" is a subset of "the people who have gold status this year."

Yet, keep in mind there are probably people who have gold status who did not fly more than 25,000 miles last year. For example, they might have gold status because of a particular credit card, by crossing the Atlantic more than six times in a year, or by being the spouse of an airline employee.

Example Three:

Let's suppose that you get a co-op with a company that does scientific & numerical computing (like Sage!), and that you're repairing some code about real-valued functions. You're taking a simple case, for debugging purposes.

$$f(x) = x^2 + 3x - 5$$

- If $x > 2$, then $f(x) > 3$.
- $x > 2$ results in $f(x) > 3$.
- $f(x) > 3$ is implied by $x > 2$.
- $x > 2$ implies $f(x) > 3$.
- $f(x) \leq 3$ implies $x \leq 2$.
- The set of "real numbers x with $x > 2$," is a subset of "the set of real numbers x with $f(x) > 3$."

Yet, keep in mind, there are situations where $f(x) > 3$ but it is not true that $x > 2$. Consider $x = -1000$ as an example.

Example Four:

Each of the following 14 statements is saying exactly the same thing.

1. If A then B.
2. B is implied by A.
3. If not B, then not A.
4. Not B implies not A.
5. All A are B.
6. B is necessary for A.
7. A is sufficient for B.
8. A requires B.
9. Not B requires not A.
10. A results in B.
11. The set of B is a superset of the set of A.
12. The set of A is a subset of the set of B.
13. A implies B.
14. B, whenever A.

Are each represented by

- if A then B.
- $A \subseteq B$.
- A Venn Diagram with the circle for A inside the circle for B.

Therefore, we can consider these "canonical forms." Every logical statement found in this packet can be represented in the "if-then" style, the subset style, or in a Venn Diagram.

It turns out the Venn Diagrams will have two or even three concentric circles. For these reasons, they are sometimes called Bullseye Diagrams.

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Level Zero:

Your objective is to just represent the situation, using the three canonical forms: subset notation, if-then notation, and a Venn Diagram/Bullseye Diagram.

- Question # 1
  - Given: J results in G
- Question # 2
  - Given: V implies J
- Question # 3
  - Given: The set of V is a subset of the set of D
- Question # 4
  - Given: K results in L
- Question # 5
  - Given: If V then A
- Question # 6
  - Given: The set of C is a superset of the set of Y

Level One:

Like level 0, your objective is to just represent the situation, using the three canonical forms: subset notation, if-then notation, and a Venn Diagram/Bullseye Diagram.

- Question # 1
  - Given: B is sufficient for G
- Question # 2
  - Given: If D then N
- Question # 3
  - Given: S requires A
- Question # 4
  - Given: C implies E
- Question # 5
  - Given: Not N requires not G
- Question # 6
  - Given: W requires D
- Question # 7
  - Given: The set of Q is a superset of the set of A
- Question # 8
  - Given: D is implied by T
- Question # 9
  - Given: M requires Q

- Question # 10
  - Given: Not S implies not J
- Question # 11
  - Given: Not K requires not D
- Question # 12
  - Given: All T are Z

Level Two:

Here, you are given two pieces of information, and you have to decide if a third piece of information is certainly true, certainly false, or not certain.

- Question # 1
  - Given #1: C implies N
  - Given #2: $x \in$ N
  - Query: Is $x \in$ C certainly true, certainly false, or not certain?
- Question # 2
  - Given #1: F , whenever D
  - Given #2: $x \notin$ F
  - Query: Is $x \in$ D certainly true, certainly false, or not certain?
- Question # 3
  - Given #1: If P then J
  - Given #2: $x \in$ P
  - Query: Is $x \in$ J certainly true, certainly false, or not certain?
- Question # 4
  - Given #1: Y , whenever L
  - Given #2: $x \in$ Y
  - Query: Is $x \in$ L certainly true, certainly false, or not certain?
- Question # 5
  - Given #1: V , whenever C
  - Given #2: $x \in$ V
  - Query: Is $x \in$ C certainly true, certainly false, or not certain?
- Question # 6
  - Given #1: Q requires G
  - Given #2: $x \notin$ G
  - Query: Is $x \in$ Q certainly true, certainly false, or not certain?
- Question # 7
  - Given #1: All W are Y
  - Given #2: $x \in$ Y
  - Query: Is $x \in$ W certainly true, certainly false, or not certain?
- Question # 8
  - Given #1: J implies L
  - Given #2: $x \in$ L
  - Query: Is $x \in$ J certainly true, certainly false, or not certain?

Level Three:

Like level two, you are given two pieces of information, and you have to decide if a third piece of information is certainly true, certainly false, or not certain.

- Question # 1
  - Given #1: Not C implies not S
  - Given #2: $x \notin$ S
  - Query: Is $x \in$ C certainly true, certainly false, or not certain?
- Question # 2
  - Given #1: Not W requires not P

  - Given #2: $x \notin$ P
  - Query: Is $x \in$ W certainly true, certainly false, or not certain?

- Question # 3
  - Given #1: Not L implies not J
  - Given #2: $x \notin$ J
  - Query: Is $x \notin$ L certainly true, certainly false, or not certain?

- Question # 4
  - Given #1: E is necessary for B
  - Given #2: $x \notin$ B
  - Query: Is $x \notin$ E certainly true, certainly false, or not certain?

- Question # 5
  - Given #1: If not N then not V
  - Given #2: $x \in$ V
  - Query: Is $x \notin$ N certainly true, certainly false, or not certain?

- Question # 6
  - Given #1: Not Y implies not P
  - Given #2: $x \notin$ P
  - Query: Is $x \notin$ Y certainly true, certainly false, or not certain?

- Question # 7
  - Given #1: A implies W
  - Given #2: $x \in$ W
  - Query: Is $x \notin$ A certainly true, certainly false, or not certain?

- Question # 8
  - Given #1: Y , whenever J
  - Given #2: $x \notin$ Y
  - Query: Is $x \notin$ J certainly true, certainly false, or not certain?

- Question # 9
  - Given #1: M implies B
  - Given #2: $x \in$ M
  - Query: Is $x \in$ B certainly true, certainly false, or not certain?

- Question # 10
  - Given #1: B implies D
  - Given #2: $x \in$ D
  - Query: Is $x \notin$ B certainly true, certainly false, or not certain?

- Question # 11
  - Given #1: The set of C is a subset of the set of M
  - Given #2: $x \notin$ M
  - Query: Is $x \in$ C certainly true, certainly false, or not certain?

- Question # 12
  - Given #1: The set of N is a superset of the set of C
  - Given #2: $x \in$ N
  - Query: Is $x \notin$ C certainly true, certainly false, or not certain?

- Question # 13
  - Given #1: Not Y requires not R
  - Given #2: $x \in$ Y
  - Query: Is $x \in$ R certainly true, certainly false, or not certain?

- Question # 14
  - Given #1: If not Y then not C
  - Given #2: $x \in$ Y
  - Query: Is $x \notin$ C certainly true, certainly false, or not certain?

- Question # 15

- Given #1: H is implied by M
- Given #2: $x \notin$ M
- Query: Is $x \notin$ H certainly true, certainly false, or not certain?

- Question # 16
  - Given #1: E is implied by C
  - Given #2: $x \notin$ C
  - Query: Is $x \notin$ E certainly true, certainly false, or not certain?

Level Four:

Like levels 0 and 1, your objective is to just represent the situation, using the three canonical forms: subset notation, if-then notation, and a Venn Diagram/Bullseye Diagram.

- Question # 1
  - Given #1: G implies R
  - Given #2: If R then N

- Question # 2
  - Given #1: The set of C is a superset of the set of N
  - Given #2: The set of V is a superset of the set of C

- Question # 3
  - Given #1: If W then Z
  - Given #2: W , whenever K

- Question # 4
  - Given #1: The set of Y is a subset of the set of R
  - Given #2: The set of Q is a superset of the set of R

- Question # 5
  - Given #1: S results in Q
  - Given #2: S , whenever G

- Question # 6
  - Given #1: C results in B
  - Given #2: All G are C

- Question # 7
  - Given #1: The set of N is a subset of the set of G
  - Given #2: The set of G is a subset of the set of P

- Question # 8
  - Given #1: K requires C
  - Given #2: The set of M is a subset of the set of K

Level Five:

Like levels 0, 1, and 4, your objective is to just represent the situation, using the three canonical forms: subset notation, if-then notation, and a Venn Diagram/Bullseye Diagram.

- Question # 1
  - Given #1: Not R implies not W
  - Given #2: All M are W

- Question # 2
  - Given #1: All V are D
  - Given #2: If E then V

- Question # 3
  - Given #1: B , whenever Z
  - Given #2: Not T implies not B

- Question # 4

  - Given #1: D is sufficient for Z
  - Given #2: Not C implies not Z

- Question # 5
  - Given #1: M implies B
  - Given #2: Q requires M

- Question # 6
  - Given #1: J is sufficient for V
  - Given #2: The set of J is a superset of the set of D

- Question # 7
  - Given #1: If H then V
  - Given #2: N results in H

- Question # 8
  - Given #1: If not P then not M
  - Given #2: P implies S

- Question # 9
  - Given #1: Z requires B
  - Given #2: C implies Z

- Question # 10
  - Given #1: All N are S
  - Given #2: Q requires N

- Question # 11
  - Given #1: The set of E is a superset of the set of Z
  - Given #2: C results in Z

- Question # 12
  - Given #1: P results in F
  - Given #2: P is implied by D

- Question # 13
  - Given #1: A is implied by C
  - Given #2: M , whenever A

- Question # 14
  - Given #1: The set of S is a superset of the set of T
  - Given #2: All Y are T

- Question # 15
  - Given #1: Not G implies not B
  - Given #2: B is implied by N

- Question # 16
  - Given #1: D is implied by R
  - Given #2: G implies R

Level Six:

You have a hypothesis, and a new data point has arrived. You have to identify if the new data point is *consistent* with the hypothesis, or a counter-example, forever shooting down the hypothesis.

- Question # 1
  - Hypothesis: P results in K
  - Given: $x \in$ K and $x \in$ P

- Question # 2
  - Hypothesis: The set of Y is a subset of the set of L
  - Given: $x \in$ Y and $x \in$ L

- Question # 3
  - Hypothesis: If L then T
  - Given: $x \in$ T and $x \in$ L

- Question # 4

  - Hypothesis: T requires R
  - Given: $x \notin$ R and $x \in$ T

- Question # 5
  - Hypothesis: S requires B
  - Given: $x \in$ S and $x \in$ B

- Question # 6
  - Hypothesis: E , whenever G
  - Given: $x \notin$ E and $x \in$ G

- Question # 7
  - Hypothesis: The set of A is a subset of the set of Y
  - Given: $x \in$ Y and $x \in$ A

- Question # 8
  - Hypothesis: The set of L is a subset of the set of Y
  - Given: $x \notin$ L and $x \in$ Y

Level Seven:

Like Level 6, you have a hypothesis, and a new data point has arrived. You have to identify if the new data point is *consistent* with the hypothesis, or a counter-example, forever shooting down the hypothesis.

- Question # 1
  - Hypothesis: If not Q then not L
  - Given: $x \notin$ L and $x \notin$ Q

- Question # 2
  - Hypothesis: J is implied by K
  - Given: $x \in$ J and $x \notin$ K

- Question # 3
  - Hypothesis: If not A then not J
  - Given: $x \in$ A and $x \in$ J

- Question # 4
  - Hypothesis: M is implied by Z
  - Given: $x \notin$ M and $x \in$ Z

- Question # 5
  - Hypothesis: Not A implies not P
  - Given: $x \in$ A and $x \in$ P

- Question # 6
  - Hypothesis: The set of Z is a subset of the set of B
  - Given: $x \notin$ B and $x \in$ Z

- Question # 7
  - Hypothesis: The set of H is a subset of the set of Y
  - Given: $x \in$ Y and $x \notin$ H

- Question # 8
  - Hypothesis: The set of B is a subset of the set of Q
  - Given: $x \notin$ Q and $x \in$ B

- Question # 9
  - Hypothesis: If not G then not Y
  - Given: $x \notin$ G and $x \in$ Y

- Question # 10
  - Hypothesis: All L are S
  - Given: $x \notin$ S and $x \notin$ L

- Question # 11
  - Hypothesis: G requires L
  - Given: $x \in$ L and $x \notin$ G

- Question # 12
  - Hypothesis: All T are F
  - Given: $x \notin$ T and $x \notin$ F

- Question # 13
  - Hypothesis: If R then D
  - Given: $x \in$ R and $x \notin$ D

- Question # 14
  - Hypothesis: Not F requires not Y
  - Given: $x \in$ Y and $x \in$ F

- Question # 15
  - Hypothesis: The set of G is a superset of the set of Q
  - Given: $x \notin$ Q and $x \in$ G

- Question # 16
  - Hypothesis: B is necessary for S
  - Given: $x \in$ S and $x \in$ B

Level Eight:

Here, you are given three pieces of information, and you have to decide if a fourth piece of information is certainly true, certainly false, or not certain.

- Question # 1
  - Given #1: L results in Z
  - Given #2: The set of F is a superset of the set of Z
  - Given #3: $x \notin$ Z
  - Query: Is $x \in$ F certainly true, certainly false, or not certain?

- Question # 2
  - Given #1: The set of K is a subset of the set of E
  - Given #2: A , whenever E
  - Given #3: $x \in$ K
  - Query: Is $x \in$ E certainly true, certainly false, or not certain?

- Question # 3
  - Given #1: H , whenever R
  - Given #2: Y , whenever H
  - Given #3: $x \in$ H
  - Query: Is $x \in$ Y certainly true, certainly false, or not certain?

- Question # 4
  - Given #1: The set of W is a subset of the set of C
  - Given #2: All C are M
  - Given #3: $x \in$ W
  - Query: Is $x \in$ C certainly true, certainly false, or not certain?

- Question # 5
  - Given #1: S implies Z
  - Given #2: All E are S
  - Given #3: $x \in$ E
  - Query: Is $x \in$ S certainly true, certainly false, or not certain?

- Question # 6
  - Given #1: B requires Y
  - Given #2: The set of R is a superset of the set of Y
  - Given #3: $x \notin$ R
  - Query: Is $x \in$ B certainly true, certainly false, or not certain?

- Question # 7
  - Given #1: If P then N
  - Given #2: The set of T is a subset of the set of P

  - Given #3: $x \notin$ T
  - Query: Is $x \in$ P certainly true, certainly false, or not certain?

- Question # 8
  - Given #1: If T then J
  - Given #2: J requires A
  - Given #3: $x \in$ A
  - Query: Is $x \in$ T certainly true, certainly false, or not certain?

- Question # 9
  - Given #1: The set of D is a subset of the set of P
  - Given #2: The set of K is a subset of the set of D
  - Given #3: $x \in$ K
  - Query: Is $x \in$ P certainly true, certainly false, or not certain?

- Question # 10
  - Given #1: G requires Q
  - Given #2: The set of Q is a subset of the set of W
  - Given #3: $x \in$ G
  - Query: Is $x \in$ W certainly true, certainly false, or not certain?

- Question # 11
  - Given #1: All V are L
  - Given #2: All K are V
  - Given #3: $x \notin$ L
  - Query: Is $x \in$ K certainly true, certainly false, or not certain?

- Question # 12
  - Given #1: W requires T
  - Given #2: T requires P
  - Given #3: $x \in$ T
  - Query: Is $x \in$ P certainly true, certainly false, or not certain?

- Question # 13
  - Given #1: N , whenever Y
  - Given #2: N implies Z
  - Given #3: $x \in$ Y
  - Query: Is $x \in$ N certainly true, certainly false, or not certain?

- Question # 14
  - Given #1: The set of S is a subset of the set of H
  - Given #2: If D then S
  - Given #3: $x \notin$ S
  - Query: Is $x \in$ D certainly true, certainly false, or not certain?

- Question # 15
  - Given #1: The set of L is a subset of the set of S
  - Given #2: The set of J is a subset of the set of L
  - Given #3: $x \notin$ S
  - Query: Is $x \in$ L certainly true, certainly false, or not certain?

- Question # 16
  - Given #1: C requires Q
  - Given #2: All S are C
  - Given #3: $x \in$ Q
  - Query: Is $x \in$ C certainly true, certainly false, or not certain?

Level Nine:

Here, you are given three pieces of information, and you have to decide if a fourth piece of information is certainly true, certainly false, or not certain.

- Question # 1
  - Given #1: If C then G
  - Given #2: Not C requires not V
  - Given #3: $x \notin$ V
  - Query: Is $x \in$ G certainly true, certainly false, or not certain?

- Question # 2
  - Given #1: K is implied by S
  - Given #2: M is implied by K
  - Given #3: $x \in$ S
  - Query: Is $x \notin$ M certainly true, certainly false, or not certain?

- Question # 3
  - Given #1: Not D requires not A
  - Given #2: A , whenever W
  - Given #3: $x \notin$ A
  - Query: Is $x \in$ D certainly true, certainly false, or not certain?

- Question # 4
  - Given #1: V is sufficient for E
  - Given #2: The set of B is a superset of the set of E
  - Given #3: $x \in$ E
  - Query: Is $x \in$ B certainly true, certainly false, or not certain?

- Question # 5
  - Given #1: The set of V is a subset of the set of C
  - Given #2: Q is necessary for C
  - Given #3: $x \notin$ Q
  - Query: Is $x \notin$ V certainly true, certainly false, or not certain?

- Question # 6
  - Given #1: H , whenever K
  - Given #2: If not K then not G
  - Given #3: $x \in$ K
  - Query: Is $x \in$ G certainly true, certainly false, or not certain?

- Question # 7
  - Given #1: V is sufficient for Z
  - Given #2: If not V then not S
  - Given #3: $x \in$ Z
  - Query: Is $x \in$ V certainly true, certainly false, or not certain?

- Question # 8
  - Given #1: The set of R is a subset of the set of H
  - Given #2: H implies S
  - Given #3: $x \in$ S
  - Query: Is $x \notin$ H certainly true, certainly false, or not certain?

- Question # 9
  - Given #1: All M are D
  - Given #2: Not N implies not D
  - Given #3: $x \notin$ M
  - Query: Is $x \in$ N certainly true, certainly false, or not certain?

- Question # 10
  - Given #1: If not D then not E
  - Given #2: E is necessary for C
  - Given #3: $x \notin$ C
  - Query: Is $x \notin$ D certainly true, certainly false, or not certain?

- Question # 11
  - Given #1: All W are R
  - Given #2: R results in V
  - Given #3: $x \in$ V
  - Query: Is $x \notin$ W certainly true, certainly false, or not certain?

- Question # 12
  - Given #1: If R then E
  - Given #2: E implies L
  - Given #3: $x \in$ R
  - Query: Is $x \in$ E certainly true, certainly false, or not certain?

- Question # 13
  - Given #1: The set of N is a subset of the set of K
  - Given #2: G results in N
  - Given #3: $x \notin$ G
  - Query: Is $x \notin$ N certainly true, certainly false, or not certain?

- Question # 14
  - Given #1: Not N requires not Z
  - Given #2: If not Z then not R
  - Given #3: $x \notin$ R
  - Query: Is $x \in$ Z certainly true, certainly false, or not certain?

- Question # 15
  - Given #1: Not T requires not W
  - Given #2: If not W then not B
  - Given #3: $x \in$ W
  - Query: Is $x \notin$ B certainly true, certainly false, or not certain?

- Question # 16
  - Given #1: If not L then not G
  - Given #2: The set of Y is a superset of the set of L
  - Given #3: $x \in$ L
  - Query: Is $x \notin$ Y certainly true, certainly false, or not certain?

- Question # 17
  - Given #1: If not D then not W
  - Given #2: P is necessary for D
  - Given #3: $x \notin$ D
  - Query: Is $x \in$ W certainly true, certainly false, or not certain?

- Question # 18
  - Given #1: Q results in Z
  - Given #2: K is necessary for Z
  - Given #3: $x \in$ Q
  - Query: Is $x \in$ Z certainly true, certainly false, or not certain?

- Question # 19
  - Given #1: All J are C
  - Given #2: N is sufficient for J
  - Given #3: $x \in$ J
  - Query: Is $x \notin$ N certainly true, certainly false, or not certain?

- Question # 20
  - Given #1: If not Z then not S
  - Given #2: If not S then not R
  - Given #3: $x \notin$ R
  - Query: Is $x \notin$ S certainly true, certainly false, or not certain?

- Question # 21
  - Given #1: J is necessary for R
  - Given #2: R is implied by L
  - Given #3: $x \in$ R

  - Query: Is $x \in$ L certainly true, certainly false, or not certain?

- Question # 22
  - Given #1: D , whenever Z
  - Given #2: K is necessary for D
  - Given #3: $x \in$ D
  - Query: Is $x \in$ K certainly true, certainly false, or not certain?

- Question # 23
  - Given #1: If R then B
  - Given #2: Not C implies not B
  - Given #3: $x \notin$ R
  - Query: Is $x \notin$ C certainly true, certainly false, or not certain?

- Question # 24
  - Given #1: J requires D
  - Given #2: The set of J is a superset of the set of R
  - Given #3: $x \in$ R
  - Query: Is $x \in$ D certainly true, certainly false, or not certain?

- Question # 25
  - Given #1: Not V implies not J
  - Given #2: V results in Q
  - Given #3: $x \notin$ J
  - Query: Is $x \in$ Q certainly true, certainly false, or not certain?

- Question # 26
  - Given #1: S is implied by G
  - Given #2: S requires Y
  - Given #3: $x \notin$ G
  - Query: Is $x \in$ S certainly true, certainly false, or not certain?

- Question # 27
  - Given #1: E , whenever F
  - Given #2: C is necessary for E
  - Given #3: $x \notin$ F
  - Query: Is $x \in$ E certainly true, certainly false, or not certain?

- Question # 28
  - Given #1: P is necessary for A
  - Given #2: Not A requires not L
  - Given #3: $x \notin$ P
  - Query: Is $x \notin$ L certainly true, certainly false, or not certain?

- Question # 29
  - Given #1: E results in G
  - Given #2: Not M requires not G
  - Given #3: $x \in$ M
  - Query: Is $x \notin$ G certainly true, certainly false, or not certain?

- Question # 30
  - Given #1: The set of E is a superset of the set of M
  - Given #2: All E are J
  - Given #3: $x \in$ J
  - Query: Is $x \notin$ E certainly true, certainly false, or not certain?

- Question # 31
  - Given #1: A requires B
  - Given #2: M is sufficient for A
  - Given #3: $x \notin$ A
  - Query: Is $x \notin$ B certainly true, certainly false, or not certain?

- Question # 32
  - Given #1: Z , whenever R

– Given #2: Z requires H
– Given #3: $x \in \mathrm{R}$
– Query: Is $x \notin \mathrm{Z}$ certainly true, certainly false, or not certain?